

# Logical Structures in Natural Language: Exercises

## Propositional Logic

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### Exercise 1: PL language

For each of the following expressions, say whether it's a well formed formula or not.

1.  $\neg(\neg p \vee q)$
2.  $p \vee (q)$
3.  $(p \rightarrow p) \rightarrow (p \rightarrow q)$
4.  $(p \vee q \wedge r)$
5.  $\neg p \wedge \neg\neg q$
6.  $\neg \wedge p$

### Exercise 2: Construction trees

(a) Build the construction tree of the following formulas  $(p \leftrightarrow r) \vee \neg q$  and  $p \leftrightarrow (r \vee \neg q)$ . For each formula, list its sub-formulae.

(b) Classify each of the following formulas as atomic, negation, conjunction, disjunction, implication or equivalence.

1.  $p \rightarrow q$
2.  $\neg p$
3.  $p$
4.  $(p \vee q) \wedge (p \vee q)$
5.  $\neg(p \rightarrow q)$
6.  $\neg(p \wedge q) \wedge \neg r$
7.  $p \rightarrow (q \wedge \neg r)$

(c) List of all the formulas that can be obtained by means of parentheses from the following string of symbols:  
 $p \wedge \neg q \rightarrow r$ .

### Exercise 3: Truth tables

Build truth tables of the following formulas:

1.  $\neg\neg A$
2.  $A \wedge (B \wedge \neg A)$
3.  $(A \rightarrow B) \rightarrow \neg B$

### Exercise 4: Tautology

Which of the following formulas are tautologies?

1.  $A \rightarrow A$
2.  $A \rightarrow (B \rightarrow A)$
3.  $(B \rightarrow A) \rightarrow A$
4.  $\neg\neg A \rightarrow A$

### Exercise 5: Equivalence

Show that the following formulas are equivalent

1.  $A \rightarrow B \equiv \neg A \vee B$
2.  $A \wedge (A \vee B) \equiv A$
3.  $A \wedge (\neg A \vee B) \equiv A \wedge B$

### Exercise 6: Translation from English into PLL

Translate the following English sentences into PL. Try to use the same structure of the sentence and give the translation keys.

Eg. If you don't sleep then you will be tired.

Translation keys:  $p$  = you sleep,  $q$  = you will be tired. Formula:  $\neg p \rightarrow q$ .

1. If it rains while the sun shines, a rainbow will appear
2. Charles comes if Elsa does and the other way around
3. Johan comes just when Peter stays at home
4. We are going, unless it is raining
5. Charles and Elsa are brother and sister or nephew and niece
6. If I have lost if I cannot make a move, then I have lost.

# Solutions

## 0.1 Exercise : Language

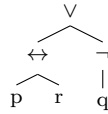
1. Yes.
2. No, but  $p \vee q$  is a wff.
3. Yes.
4. No, but  $(p \vee q) \wedge r$  and  $p \vee (q \wedge r)$  are wff.
5. Yes.
6. No.

## 0.2 Exercise : Construction trees

(a)

- $(p \leftrightarrow r) \vee \neg q$ .

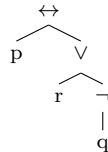
**Construction Tree**



**Subformula:**  $p, r, q, \neg q, p \leftrightarrow r, (p \leftrightarrow r) \vee \neg q$ .

- $p \leftrightarrow (r \vee \neg q)$ .

**Construction Tree**



**Subformula:**  $p, r, q, \neg q, r \vee \neg q, p \leftrightarrow (r \vee \neg q)$ .

(b)

1. implication
2. negation
3. atomic formula
4. conjunction
5. negation
6. conjunction
7. implication

(c) 1.  $(p \wedge \neg q) \rightarrow r$ , 2.  $p \wedge (\neg q \rightarrow r)$ , 3.  $p \wedge \neg(q \rightarrow r)$ .

### 0.3 Exercise : Truth Tables

1.  $\neg\neg A$

$A$	$\neg$	$\neg$	$A$
T	T	F	F
F	F	T	T

(1)

2.  $A \wedge (B \wedge \neg A)$

$A$	$B$	$A$	$\wedge$	$(B$	$\wedge$	$\neg$	$A)$
T	T	F	F	F	F	F	F
T	F	F	F	F	F	F	F
F	T	F	F	T	T	T	T
F	F	F	F	F	F	T	T

(1)

3.  $(A \rightarrow B) \rightarrow \neg B$

$A$	$B$	$(A$	$\rightarrow$	$B)$	$\rightarrow$	$\neg$	$B$
T	T	T	T	F	F	F	F
T	F	F	F	T	T	T	T
F	T	T	F	F	F	F	F
F	F	T	T	T	T	T	T

(1)

### 0.4 Exercise : Tautologies

1.  $A \rightarrow A$ . Tautology.

$A$	$A$	$\rightarrow$	$A$
T	T	T	T
F	F	T	T

(1)

2.  $A \rightarrow (B \rightarrow A)$ . Tautology.

$A$	$B$	$A$	$\rightarrow$	$(B$	$\rightarrow$	$A)$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

(1)

3.  $(B \rightarrow A) \rightarrow A$

$A$	$B$	$(B$	$\rightarrow$	$A)$	$\rightarrow$	$A$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	F	T	T	T
F	F	T	T	F	F	F

(1)

4.  $\neg\neg A \rightarrow A$ . Tautology.

$A$	$\neg$	$\neg$	$A$	$\rightarrow$	$A$
T	T	F	T	T	T
F	F	T	T	T	T

(1)

### 0.5 Exercise : Equivalence

1.  $A \rightarrow B$  and  $\neg A \vee B$ . Yes

$A$	$B$	$(A \rightarrow B)$	$\neg A \vee B$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T
		(1)	(2)

2.  $A \wedge (A \vee B)$  and  $A$ . Si.

$A$	$B$	$A \wedge (A \vee B)$
T	T	T
T	F	T
F	T	F
F	F	F
		(1)                      (2)

3.  $A \wedge (\neg A \vee B)$  and  $A \wedge B$ . Yes

$A$	$B$	$A \wedge (\neg A \vee B)$	$A \wedge B$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F
		(1)	(2)

### 0.6 Exercise: Translation

1.  $p$  = it rains,  $r$  = the sun shines,  $q$  = a rainbow will appear  
 $(p \wedge r) \rightarrow q$
2.  $p$  = Charles comes,  $q$  = Elsa comes  
 $(q \rightarrow p) \wedge (p \rightarrow q)$  or  $(p \leftrightarrow q)$
3.  $p$  = Johan comes,  $q$  = Peter stays at home  
 $p \leftrightarrow q$
4.  $p$  = We are going,  $q$  = it rains  
 $\neg p \leftrightarrow q$
5.  $p$  = Charles and Elsa are brother and sister,  $q$  = Charles and Elsa are nephew and niece.  
 $p \vee q$
6.  $p$  = I can make a move,  $q$  = I have lost.  
 $(\neg p \rightarrow q) \rightarrow q$