1 Context Free Grammar

1.1 CF Language

Let G1, G2 and G3 be three grammars consisting of the terminal symbols $\{a, b\}$, the non-terminal symbols $\{A, B\}$, the start symbol S, and of the following rewriting rules

G1 $S \to AB, A \to ab, B \to bb$
G2 $S \to AB, S \to AA, A \to aB, A \to ab, B \to b$
G3 $S \to aS, S \to Sb, S \to$

(a) find the languages generated by the three grammars. (b) Construct a parse tree of a^2b^3 using G3.

1.2 CF Grammars

Construct a CFG generating the following language:

$L = a^m b^n a^m \quad (0 \leq n, m)$

1.3 CFG and Natural Language

Starting from the following rewriting rules

- $PN \to$ John | Mary | London | They
- $IV \to$ travel
- $TV \to$ saw | see | like
- $DET \to$ the | a
- $N \to$ flower | city | man | telescope
- $ADJ \to$ red | nice
- $AUX \to$ will
- $P \to$ with | to

(a) write a CFG that recognizes the sentences below. Try to use as few rules as possible. (b) Construct their parse trees. If the sentence is ambiguous, make sure your grammar builds all possible parse trees.

1. They like a red flower.
2. Mary will travel to London.
3. Mary will travel to the city
4. John saw the man with the telescope.
5. The man will see Mary

2 Categorial Grammar and Lambda Calculus

(a) Build the categorial grammar lexicon needed to parse the following sentences

1. Mary likes John
2. A student likes John
3. A student who Mary knows left

(b) Give their parse trees. (c) Using CG rules labelled with lambda-terms, build their meaning representations.

3 Vectors

a) Given the vector $\vec{v} = (1, 1, 2)$ and $\vec{w} = (3, 4, 5)$, compute their dot product: $\vec{v} \cdot \vec{w}$.

b) Given the vector $\vec{v}$ in (a) compute its length.

c) Find the unit vector, $\vec{u}$, in the same direction of $\vec{v}$ given in (a).

Recall their definition:

$$\vec{v} \cdot \vec{w} = \sum_{i=1}^{n} v_i w_i$$

$$||\vec{v}|| = \sqrt{\sum_{i=1}^{n} v_i^2}$$

$$\vec{u} = \frac{\vec{v}}{||\vec{v}||}$$