Lambda Terms

Raffaella Bernardi

22-10-2020

1 Exercise 1: From Set to Functional Perspective

Look at the Knowledge Base below.

1. Harry is a wizard.
2. Hagrid scares Dudley.
3. All wizards are magical.
4. Uncle Vernon hates anyone who is magical.
5. Aunt Petunia hates anyone who is magical and scares Dudley.

Build a model for it by (i) writing your interpretation for $wizards, magical, scares, hates$ using the relational interpretation first, and then the functional one.

2 Exercise 2: Well formed formula

Let j be a constant of type e; M of type $e \to t$; S of type $((e \to t) \to (e \to t))$, and P of type $(e \to t) \to t$. Furthermore, x is a variable of type e, and Y a variable of type $(e \to t)$.

Determine which of the following is well-formed, give its type.

1. $(\lambda x.M(x))(P)$.
2. $(\lambda x.M(x))(j)$.
3. $\lambda x.M(j)$.
4. $S(\lambda x.M(x))$.
5. $(\lambda Y.Y(j))(M)$
6. $\lambda x.(M(x) \wedge M(j))$
7. $(\lambda x.M(x)) \wedge M(j)$

3 Exercise 3: β-conversion

Let j be a constant of type e; M of type $(e \to t)$, and A of type $e \to (e \to t)$. Furthermore, x and y are variables of type e, and Y is a variable of type $e \to t$. Reduce the following expression as much as possible by means of β-conversion.

1. $\lambda x(M(x))(j)$
2. $\lambda Y(Y(j))(M)$
3. $\lambda x \lambda Y(Y(x))(j)(M)$
4. $\lambda x \forall y(A(x)(y))(j)$
5. $\lambda x \forall y(A(x)(y))(y)$
6. $\lambda Y(Y(j))\lambda x(M(x))$
7. $\lambda Y \forall x(Y(x))\lambda y(A(x)(y))$

4 Solutions

4.1 Exercise 1

$$
\begin{array}{lcll}
[\![wizard]\!] & = & \{harry\} & \{x|wizard(x) = 1\} \\
[\![magical]\!] & = & \{harry\} & \{x|magical(x) = 1\} \\
[\![scares]\!] & = & \{(hagrid, dudley)\} & \{(x, y)|(scares\ (y))\ (x)\} \\
[\![hates]\!] & = & \{(vernon, harry)\} & \{(x, y)|(hates\ (y))\ (x)\}
\end{array}
$$

4.2 Exercise 2

1. $(\lambda x.M(x))(P)$. [NWF]
2. $(\lambda x.M(x))(j)$. [WF]
3. $\lambda x.M(j)$. [NWF: vacus abstraction]
4. $S(\lambda x.M(x))$. [WF]
5. $(\lambda Y.Y(j))(M)$ [WF]
6. $\lambda x.(M(x) \wedge M(j))$ [WF]
7. $(\lambda x.M(x)) \wedge M(j))$ [NWF: $\lambda x.M(x)$ and $M(j)$ are of types $e \rightarrow t$ and t, resp. $\wedge$ coordinates terms of types t]

4.3 Exercise 3

1. $\lambda x(M(x))(j)$ $[M(j)]$
2. $\lambda Y(Y(j))(M)$ $[M(j)]$
3. $\lambda x \lambda Y(Y(x))(j)(M)$ $[M(j)]$
4. $\lambda x \forall y(A(x)(y))(j)$ $[\forall y.A(j)(y)]$
5. $\lambda x \forall y(A(x)(y))(y)$ $[\forall y.A(z)(y)]$
6. $\lambda Y(Y(j))\lambda x(M(x))$ $[M(j)]$
7. $\lambda Y \forall x(Y(x))\lambda y(A(x)(y))$ $[\forall z.A(x)(z)]$