1. Logic & Language

- The meaning of a sentence is its **truth value**. (Tarski)
- The meaning of a sentence is obtained **compositionaly** starting from the meaning of the words. (Frege)
- The meaning of word is captured by **Set Theory**

Question: Can we capture formally the relation between **Sintax and Semantic** of Natural Language?

1.1. Set Theory: Lexicon

Interpretation function

$$
\begin{array}{lcl}
[\![\texttt{tommaso}]\!] & = & \text{tommy;} \\
[\![\texttt{valerio}]\!] & = & \text{valerio;} \\
[\![\texttt{roberto}]\!] & = & \text{roby;} \\
[\![\texttt{raffaella}]\!] & = & \text{raffa;} \\
[\![\texttt{student}]\!] & = & \{\text{tommy, valerio}\}; \\
[\![\texttt{teacher}]\!] & = & \{\text{roby}\}; \\
[\![\texttt{italian}]\!] & = & \{\text{tommy, valerio, raffa, roby}\}; \\
[\![\texttt{speak}]\!] & = & \{\text{raffa}\}. \\
[\![\texttt{listen}]\!] & = & \{\text{tommy, valerio, roby}\}; \\
[\![\texttt{know}]\!] & = & \{\langle\text{roby, raffa}\rangle, \langle\text{raffa,roby}\rangle\};
\end{array}
$$

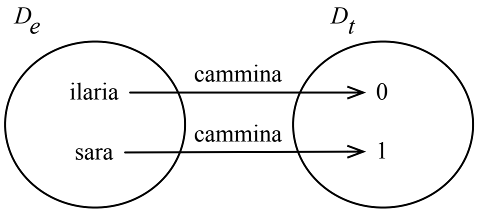
Which is the meaning of “Every student?”

$$
\begin{array}{lcl}
[\![\texttt{Every student}]\!] & = & \{X \subseteq E \mid [\![\texttt{student}]\!] \subseteq X\} \\
 & = & \{\{\text{tommy, valerio}\}, \{\text{tommy, valerio, raffa, roby}\}, \{\text{tommy, valerio, roby}\}\} \\
 & = & \{[\![\texttt{student}]\!], [\![\texttt{italian}]\!], [\![\texttt{listen}]\!]\}
\end{array}
$$

ie. a set of properties.

Question: How is this reflected at the syntactic level? Which is their CG syntatic category?

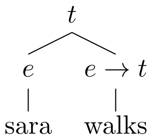
1.2. From sets to Functions



$$[\![\texttt{Sara}]\!] = \text{sara}$$
$$[\![\texttt{Ilaria}]\!] = \text{ilaria}$$
$$[\![\texttt{Cammina}]\!] = \{x \mid Cammina(x) = 1\} = \{\text{sara}\}$$

i.e. the caratheristic function of the set.

Words can be seen as functions. As incomplete sentences, they miss something to form a sentence.



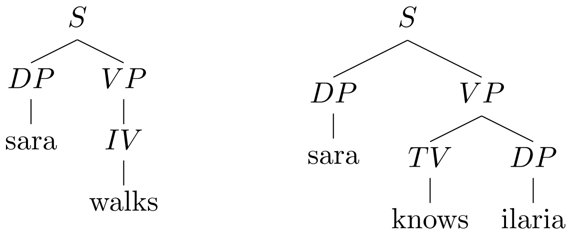
Domains of denotation Words denote in domains of different types. E.g. Entities denote in the domain of entities (D_e), sentences in the domain of truth value (D_t), intransitive verb into the domain of functions from $D_e \to D_t$.

2. Mapping: categories-types

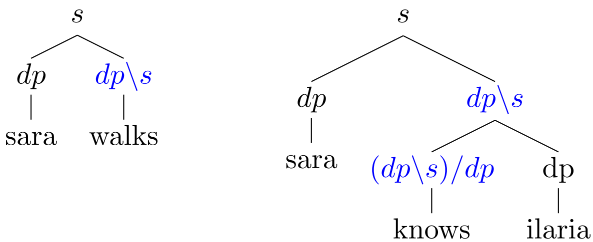
To set up the form-meaning correspondence, it is useful to build a language of semantic types in parallel to the syntactic type language.

2.1. Syntactic Categories

CFG:



Nuovo:



Categorial Grammar: more complex categories than CFG.

2.2. Types

Definition 2.1 (Types) Given a non-empty set of basic types Base, the set of types TYPE is the smallest set such that

i. $\mathsf{Base} \subseteq \mathsf{TYPE}$;
ii. $(a \to b) \in \mathsf{TYPE}$, if a and $b \in \mathsf{TYPE}$.

Note that this definition closely resembles the one of the syntactic categories of CG. The only difference is the lack of directionality of the functional type $(a \to b)$.

2.3. Mapping

A function mapping the syntactic categories into TYPE can be given as follows.

Definition 2.2 (Categories and Types) *Let us define a function* $\texttt{type} : \mathsf{CAT} \to \mathsf{TYPE}$ *which maps syntactic categories to semantic types.*

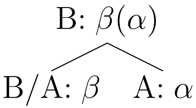
$$\begin{array}{ll} \texttt{type}(np) = e; & \texttt{type}(A/B) = (\texttt{type}(B) \to \texttt{type}(A)); \\ \texttt{type}(s) = t; & \texttt{type}(B\backslash A) = (\texttt{type}(B) \to \texttt{type}(A)); \\ \texttt{type}(n) = (e \to t). & \end{array}$$

Syntactic Rules and Lambda calcuslus The syntact rules of CG corresponds to the lamda calculus rules.

Thanks to this correspondence, we can build meaning representation of sentences while building the syntactic tree, simply by apply the labeled syntactic rules (next slides).

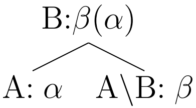
If you assign to your lexical entry a category and type that miss-match, you will not be able to build the meaning representation of the sentence while building its syntactic CG tree.

3. CG: syntactic rules labeled with terms



$$B/A : \beta,\ A : \alpha \Rightarrow B : \beta(\alpha)$$

$$\frac{B/A \quad A}{B}\ (/\mathrm{E})$$



$$A : \alpha,\ A\backslash B : \beta \Rightarrow B : \beta(\alpha)$$

$$\frac{A : \alpha \quad A\backslash B : \beta}{B : \beta(\alpha)}\ (\backslash\mathrm{E})$$

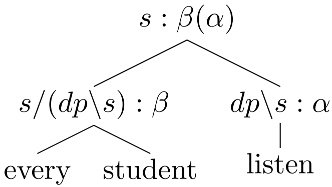
3.1. Sintassi-Semantica: Quantifiers

$$
\begin{aligned}
[\![\texttt{every student}]\!] &= \{X \subseteq E \mid [\![\texttt{student}]\!] \subseteq X\} \\
&= \{[\![\texttt{student}]\!], [\![\texttt{italian}]\!], [\![\texttt{listen}]\!]\}
\end{aligned}
$$

Il QP is interpreted as a function: $D_{(e \to t)} \to D_t$

Which is the corresponding syntactic category?

$$s/(dp\backslash s)$$



assign the meaning representation to the leaves of the tree, by applying the rules you build the meanig representation of the sentence.

Above I have written β and α, replace them with the lambda term for the words in the leave.

3.2. Syntax-Semantics: Relative Pronoun

$$(n\backslash n)/(s/np)$$

How is this reflected on the semantics?

The relative pronoun is a function that takes first a property and then a noun.

$$[\![\texttt{book which sara wrote}]\!] = [\![\texttt{book}]\!] \cap [\![\texttt{sara wrote}]\!]$$

Which is the semantic type? Can you build the meaning representation of the phrase "book which sara wrote" by labeling the corresponding CG syntactic tree?