Computational Linguistics: Formal Semantics

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1. Admin

How is it going with the recorded classes? It seems they are not really used. Should I dowload the files with shared screen only and upload them in Moodle instead?

The files in the link I have shared are automatically deleted after 14 days. The first ones are about to be deleted.

Friday 23rd of October, shall we meet all online? (I need to use the whiteboard).

From next week, you are going to have class on Fridays always with Luca (to let you practice what you are learning with him.) Hence, we cannot have our class on Friday 6th of Nov. at 11:00-12:30. Shall we have it online earlier or later in the day?

2. The pillers of Formal Semantics

- Tarski
- Frege
- Montague

2.1. Reminder Tarski: What does a given sentence mean? (video)

The meaning of a sentence is its truth value.

"Snow is white" is true iff snow is white.

Rephrased in: "Which is the meaning representation of a given sentence to be evaluated as true or false?"

- **Meaning Representations**: Predicate-Argument Structures are a suitable meaning representation for natural language sentences. E.g. the meaning representation of "Lori knows Alex" is $\mathtt{know}(\mathtt{lori}, \mathtt{ale})$

  whereas the meaning representation of "A student knows Alex" is $\exists x.\mathtt{student}(x) \wedge \mathtt{knows}(x, \mathtt{ale})$.

- **Interpretation**: a sentence is taken to be a proposition and its meaning is the truth value of its meaning representations. E.g.

  $[\![\exists x.\mathtt{student}(x) \wedge \mathtt{walk}(x)]\!] = 1$ iff standard FOL definitions are satisfied.

2.2. Reminder Frege: Quantifiers (video)

FOL quantifiers Frege introduced the FOL symbols: $\exists$ and $\forall$ to represent the meaning of quantifiers ("some" and "all") precisely and to avoid ambiguities.

Natural Language Syntax-Semantics The grammatical structure:

> "A natural number is bigger than all the other natural numbers."

can be represented as:

1. $\forall x \exists y Bigger(y, x)$ — true
2. $\exists y \forall x Bigger(y, x)$ — false

Hence, there can be a mismatch between syntactic and semantics representations

3. Reminder: Montague Formal semantics (video)

The foundational work by Frege, Carnap, and Tarski had led to a rise in work on modal logic, tense logic, and the analysis of **philosophically interesting issues in natural language**. Philosophers like Kripke and Hintikka added model theory.

These developments went hand-in-hand with the **logical syntax** tradition (Peirce, Morris, Carnap), distinguishing syntax (well-formedness) from semantics (interpretation) and pragmatics (use).

Though the division was inspired by language, **few linguists attempted to apply the logician's tools in linguistics as such**.

This changed with **Montague**.

> "I reject the contention that an important theoretical difference exists between formal and natural languages." (Montague, 1974)(p.188)

A compositional approach, using a "rule-by-rule" translation (Bach) of a syntactic structure into a first-order, intensional logic. This differed substantially from transformational approaches (generative or interpretative semantics).

3.1. Key: Semantics is model-theoretic

The focus is on meaning as "extension":

> "The extension of an expression is the set of things it extends to, or applies to" (Wikipedia)

Ingredients:

- A model of the world
- the model consists of sets
- words in a language refer or denote parts of the model
- a proposition is true iff it corresponds to state of affairs in the model.

Hence, Formal Semantics is also called **Denotational Semantics**

4. Reminder: Propositional Logic (PL)

A **model** consists of two pieces of information:

- which collection of atomic propositions we are talking about (**domain**, D),
- and for each formula which is the appropriate **semantic value**, this is done by means of a function called **interpretation function** ($\mathcal{I}$).

Thus a model $\mathcal{M}$ is a pair: $(D, \mathcal{I})$.

Propositional Logic (PL): represents **propositions**. Atomic ones, p, q, r and complex ones built with truth-functional connectives: $P \wedge Q, P \vee Q, P \rightarrow Q, \neg P$.

4.1. How far can we go with PL?

1. Casper is bigger than John
2. John is bigger than Peter
3. Therefore, Casper is bigger than Peter.

Questions:

How would you formalize this inference in PL?

What do you need to express that cannot be expressed in PL?

Answer:

You need to express: “relations” (is bigger than) and “entities” (Casper, John, Peter)

4.2. What else do we need?

1. Bigger(casper,john)
2. Bigger(john,peter)
3. Therefore, Bigger(casper,peter)

Question: Do you still miss something?

The knowledge that: for all x, for all y and for all z

IF Bigger(x,y) AND Bigger(y,z) THEN Bigger(x,z).

We miss the universal quantifier: $\forall$.

4.3. More expressive logic: First order Logic

- Just like in propositional logic, a (complex) FOL formula may be true (or false) with respect to a given interpretation.
- An interpretation specifies referents for
  constant symbols $\rightarrow$ **objects**
  predicate symbols $\rightarrow$ **relations**
- An atomic sentence $P(t_1, \ldots, t_n)$ is true in a given interpretation
  iff the *objects* referred to by $t_1, \ldots, t_n$
  are in the *relation* referred to by the predicate P.

4.4. Variables and Quantifiers

"All A are B" can be read as saying:

For all x, **if** x is A **then** x is B.

i.e. all the members of A are also members of B, i.e A is included in B, $A \subseteq B$.

We write this as: $\forall x.A(x) \to B(x)$

"Some A are B" can be read as saying:

For some x, x is A **and** x is B.

i.e. there exists at least a members of A that is also a members of B.

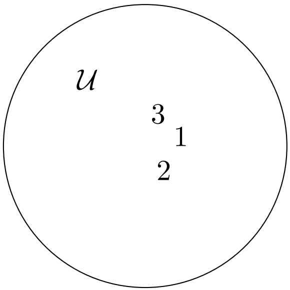
We write this as: $\exists x.A(x) \wedge B(x)$

5. Meaning as Reference: constants

Following Tarski, we build a Model by looking at a Domain (the set of entities) and at the **interpretation function** $\mathcal{I}$ which assigns an appropriate **denotation** in the model $\mathcal{M}$ to each individual and n-place predicate constant.

Individual constants If α is an individual constant, $\mathcal{I}$ maps α onto one of the entities of the universe of discourse $\mathcal{U}$ of the model $\mathcal{M}: \mathcal{I}(\alpha) \in \mathcal{U}$. The meaning of all the other words is based on the entities.

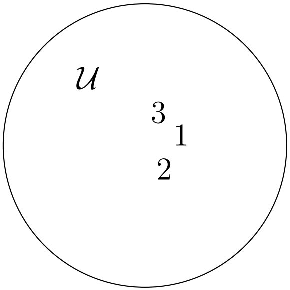
Take the constants: a, b, c and the Universe consisting of the entities 1, 2, 3:



$$\mathcal{I}(a) = 1,\ \mathcal{I}(b) = 2,\ \mathcal{I}(c) = 3$$

5.1. Meaning as Reference: properties

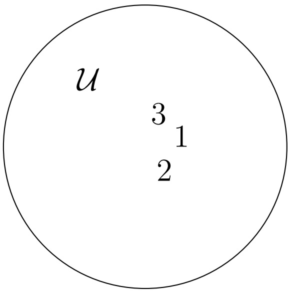
Set of entities the property of being "odd" denotes the **set of entities** that are "odd". Formally, for O (res. E) a one-place predicate, the interpretation function $\mathcal{I}$ maps O onto a subset of the universe of discourse $\mathcal{U} : \mathcal{I}(P) \subseteq \mathcal{U}$.



$\mathcal{I}(O) = \{1, 3\}$ $\mathcal{I}(E) = \{2\}$

5.2. Meaning as Reference: relation

Set of entities pairs The relation such as "bigger" denotes **sets of ordered pairs of entities**, namely all those pairs which stand in the "bigger" relation. Given the relation R, the interpretation function $\mathcal{I}$ maps R onto a set of ordered pairs of elements of $\mathcal{U}$: $\mathcal{I}(R) \subseteq \mathcal{U} \times \mathcal{U}$



$$\mathcal{I}(B) = \{(2,1),(3,2),(3,1)\}$$

5.3. Meaning as Reference: Linguistic example

Let $[\![\texttt{w}]\!]$ indicate the interpretation of **w** $(\mathcal{I}(w))$:

$$
\begin{array}{lcl}
[\![\texttt{sara}]\!] & = & \text{sara}; \ldots \\
[\![\texttt{walk}]\!] & = & \{\text{lori}\}; \\
[\![\texttt{know}]\!] & = & \{(\text{lori, alex}), (\text{alex,lori}), (\text{sara, lori}), \\
 & & (\text{lori, lori}), (\text{alex, alex}), (\text{sara, sara}), (\text{pim, pim})\}; \\
[\![\texttt{student}]\!] & = & \{\text{lori, alex, sara}\}; \\
[\![\texttt{professor}]\!] & = & \{\text{pim}\}; \\
[\![\texttt{tall}]\!] & = & \{\text{lori, pim}\}.
\end{array}
$$

which is nothing else to say that, for example, the relation **know** is the **set of pairs** (α, β) where α knows β; or that 'student' is the set of all those elements which are a student.

5.4. Exercise

1. Harry is a wizard.
2. Hagrid scares Dudley.
3. All wizards are magical.
4. Uncle Vernon hates anyone who is magical.
5. Aunt Petunia hates anyone who is magical and anyone who scares Dudley.

Build a model for it by writing your interpretation for wizards, magical, scares, hates using the set theoretical interpretation.

$$
\begin{array}{lcl}
[\![\texttt{Harry}]\!] & = & \mathrm{h}; \ldots \\
[\![\texttt{Hagrid}]\!] & = & \mathrm{ha}; \\
[\![\texttt{Vernon}]\!] & = & \mathrm{v}; \\
[\![\texttt{Petunia}]\!] & = & \mathrm{p}; \\
[\![\texttt{Dudley}]\!] & = & \mathrm{d}; \\
[\![\texttt{wizard}]\!] & = & \{\mathrm{h}\}; \\
[\![\texttt{magical}]\!] & = & \{\mathrm{h}\}; \\
[\![\texttt{hates}]\!] & = & \{(\mathrm{v},\ \mathrm{h}), (\mathrm{p},\mathrm{h}), (\mathrm{p},\ \mathrm{ha})\}
\end{array}
$$

6. From sets to functions

A set and **its characteristic function** amount to the same thing:
if f_X is a function from Y to $\{F, T\}$, then $X = \{y \mid f_X(y) = T\}$.

In other words, the assertion '$y \in X$' and '$f_X(y) = T$' are equivalent.

$$[\![student]\!] = \{\text{lori, alex, sara}\}$$

`student` can be seen as a function from entities to truth values:

$$[\![student]\!] = \{x | \texttt{student}(x) = T\}$$

Back to this tomorrow.

7. Domain and Interpretation

- Socrates, Plato, Aristotle are philosophers
- Mozart and Beethoven are musicians
- All of them are human beings
- Socrates knows Plato.
- Mozart knows Beethoven.

Which do you thing is the Universe or Domain of discourse?

Are the statements below true or false in the above situation?

1. $\forall x.HumanBeings(x)$?
2. $\exists x.HumanBeings(x) \land Musicians(x)$?
3. $\forall x.Female(x) \to Musicians(x)$?

7.1. Sub-formulas, free and bound

Sub-formula is a string inside a wff such that it is also a wff. For instance, in the wff: $\forall x(P(x) \wedge Q(y))$ the sub-formulas are:

- $\forall x(P(x) \wedge Q(y))$
- $P(x) \wedge Q(y)$
- $P(x)$
- $Q(y)$

Quantifier It is always before a sub-formula (its scope) and it is associated to a variable (the variable appearing just after the quantifier and of which the quantifier express its quantification.)

Variable A variable inside a formula is said to be **free** if it does not occur in any sub-formula preceded by a quantifier associated to such variable. Otherwise, it is said to be **bound**. In the example above x is bound whereas y is free. A formula is said to be **closed** if it does not contain free variable, it is said to be **open** if it contains free variable.

7.2. Exercises on FoL syntax

1. All lectures teach Logic.

   $\forall x.\texttt{Lectures}(x) \to \texttt{Teach}(x, \texttt{logic})$

2. Not all lecturers teach Logic.

   $\neg(\forall x.\texttt{Lectures}(x) \to \texttt{Teach}(x, \texttt{logic}))$

3. Some lectures teach logic

   $\exists x.\texttt{Lectures}(x) \wedge \texttt{Teach}(x, \texttt{logic})$

4. Some lectures do not teach Logic

   $\exists x.\texttt{Lectures}(x) \wedge \neg\texttt{Teach}(x, \texttt{logic})$

Question: Are any of these sentences/formulas equivalent?

Yes: 2 and 4!

Question: What do you conclude? How can you generalize this claim?

$\neg\forall x.A = \exists x \neg A$

8. Formal Semantics: Summing up

Aim: Specify semantic representations for the **lexical items** based on reference and build the representation of sentence **compositionally**.

Solution We have seen that lexical meaning can be represented by sets or equivalently by functions.

Next time we are going to speak of compositionality.